EFFECTIVE IMPLEMENTATION OF M-D MULTIRATE SYSTEMS BY FACTORIZATION OF M-D POLYNOMIAL POLYPHASE MATRICES

Tchobanou M.K.*, Woodburn C. J.**

* Moscow Power Engineering Institute
105835, GSP, Moscow E-250, 14 Krasnokazarmennaya st.
Tel: +7 (095) 362 7463, E-mail: tmk@vvk2.mpei.ac.ru
** Pittsburg State University, Department of Mathematics
1701 S. Broadway, Pittsburg, KS 66762, USA
Tel. (620) 235-4409, E-mail: cwoodbur@pittstate.edu

Abstract: Factorization of multi-dimensional (M-D) polynomial matrices is an effective tool which finds different applications. One of them is effective implementation of M-D multirate systems. The factorization significantly reduces the number of computations and allows one to design filter banks with 'good' coefficients (from implementation point of view).

1. Introduction

There are different ways to factor M-D polynomial matrices. For example N.K. Bose, C. Charoenlarpnopparut, and Z. Lin have looked at factoring into two factors where one is prime in some sense [1, 3, 4]. Guiver-Bose showed how to implement primitive factorization of bivariate matrices using computations in the ground field only. Putting a matrix into the Smith normal form yields a factorization into a product of elementary matrices and a diagonal matrix. Factorization into elementary matrices might be based on Suslin's Stability Theorem (SST).

SST states that theoretically every 3x3 or larger square polynomial matrix with determinant that equals one can be factored into product of elementary matrices. An algorithm by H. Park and C. Woodburn exists for SST, but the algorithm is not practical, has not yet implemented, and yields many factors [5, 6]. The local case subalgorithm of H. Park and C. Woodburn can sometimes be applied to yield a polynomial solution. It has been implemented in Maple and progress is being made in improving the implementation. Other heuristic methods, for example one modeled after Gaussian Elimination, sometimes work.

Techniques may also be applied to some polynomial matrices with determinant not equal to one to yield factorizations with "mostly" elementary matrix factors [9]. Such a technique is given in this paper.

2. Factorization of M-D polynomial polyphase matrices

Any ***M***-channel filter bank is represented by ***MxM*** polyphase polynomial matrices [2, 7, 8, 10]. The polyphase matrix may be factored into a product of elementary and diagonal matrices by application of a Gaussian elimination procedure (an elementary matrix $\mathbf{e}_{ij}(f)$ is a matrix which coincides with the identity except for possibly a single off-diagonal entry f in the ij-position).

The main reasons behind the factorization of the polyphase matrices are: to reduce the number of required computations (additions, multiplications) and to obtain 'good' coefficients (integers, powers of two and so on) for the filters.

As a result, the following factorization in **2-D** case (for N = 1 and M = 1 - see [8, 9]) was obtained:

$$\mathbf{H_p} = \frac{1}{2}\begin{bmatrix} 1 & B \\ B & C \end{bmatrix} = \mathbf{H_1}\cdot\mathbf{H_2}\cdot\mathbf{H_3}, \text{ where } \mathbf{H_1} = \begin{bmatrix} 1 & 0 \\ B & 1 \end{bmatrix}, \quad \mathbf{H_2} = \begin{bmatrix} 1 & 0 \\ 0 & -2\cdot ab \end{bmatrix}, \quad \mathbf{H_3} = \begin{bmatrix} 1 & B \\ 0 & 1 \end{bmatrix}$$

and $B = 1/4\cdot(1+b)(1+a),\ (a,b) = \mathbf{z}^{\mathbf{V}_Q} = (z_1 z_2, z_1 z_2^{-1})$. Similar results were obtained for $N = M = 2$ and $N = M = 3$. It should be mentioned that this procedure may be applied for any values of N and M.

In the **3-D** case the result is:

$$\mathbf{H_p} = \frac{1}{2}\begin{bmatrix} 1 & B \\ B & C \end{bmatrix} = \mathbf{H_1}\cdot\mathbf{H_2}\cdot\mathbf{H_3}, \text{ where } \mathbf{H_1} = \begin{bmatrix} 1 & 0 \\ B & 1 \end{bmatrix}, \quad \mathbf{H_2} = \begin{bmatrix} 1/2 & 0 \\ 0 & -ad \end{bmatrix}, \quad \mathbf{H_3} = \begin{bmatrix} 1 & B \\ 0 & 1 \end{bmatrix},$$

and

$$B = -1/16\cdot((d+a)(ad+1)(a+b^2 d) - 4ad(b^2 d + bd + abd + b + ab + a))/abd,$$

$(a,b,d) = \mathbf{z}^{\mathbf{V}_{FCO}} = (z_1 z_2^{-1}, z_2^{-1} z_3^{-1}, z_1 z_2)$. It can be noticed that in both the 2-D and 3-D cases the coefficients are powers of two.

3. Comparison of operations number

The necessary numbers of computations for both – non-factorized and factorized cases - are given in tables 1 and 2 below. It is evident that the factorization of the polyphase polynomial matrices has a really big impact on the computation speed (see fig. 1). The fact that the coefficients of the multipliers may be powers of two is also quite important. In the tables * denotes NO factorization and ** - FACTORED.

N	**M**	Additions		Multiplic.		Operations		**Gain**
		*	**	*	**	*	**	*/**
1	1	14	**6**	8	**3**	22	**9**	**2,44**
2	2	50	**14**	34	**5**	84	**19**	**4,42**
3	3	110	**30**	110	**29**	220	**59**	**3,73**
4	4	194	**54**	194	**53**	388	**107**	**3,63**
5	5	302	**86**	302	**85**	604	**171**	**3,53**

Table 1: Comparison of non-factored and factored polyphase matrices (2-D case)

N	**M**	Additions		Multiplic.		Operations		**Gain**
		*	**	*	**	*	**	*/**
1	1	90	**28**	70	**15**	160	**43**	**3,72**
2	2	480	**124**	412	**79**	892	**203**	**4,39**
3	3	1232	**144**	1236	**147**	2468	**291**	**8,48**

Table 2: Comparison of non-factored and factored polyphase matrices (3-D case)

Conclusions

The method for the design of 2-D and 3-D FBs with the desired properties based on polynomial approaches are given. Bernstein polynomials allow one to design analytically the polyphase polynomial matrices. The factorization of these matrices speeds up the computation rate.

References

1. N.K. Bose, C. Charoenlarpnopparut. Multidimensional FIR filter bank design using Gröbner bases. *IEEE Trans. Circ., Syst. II: Analog and Digital Signal Processing*, 46:1475–1486, December 1999.
2. T. Cooklev, A. Nishihara, Y. Toshiyuki, M. Sablatash. Multidimensional two-channel linear phase FIR filter banks and wavelet bases with vanishing moments. *Multidimens. Syst. and Sign. Proc.*, 9:39–76, 1998.
3. W. Lawton, Z. Lin. Matrix completion problems in multidimensional systems. In *Proc. IEEE Int. Symp. Circ. and Syst.*, 1999.
4. Z. Lin, N. Bose. Some conjectures on multivariate polynomial matrices. In *Proc. The 2nd International Workshop on Multi-dimensional (nD) Systems NDS-2000*, pages 165–170, Zielona Góra, Poland, 2000.
5. H. Park, C.J. Woodburn. An algorithmic proof of Suslin's stability theorem for polynomial rings. *Journ. of Algebra*, 178:277–298, 1995.
6. A.A. Suslin. On the structure of the special linear group over polynomial rings. *English translation in Math. USSR Izvestija*, 11:221–238, 1977.
7. M. Tchobanou. Polynomial methods for multidimensional filter banks' design. In *Proc. 10th European Signal Processing Conference EUSIPCO- 2000*, Tampere, Finland, 2000.
8. M. Tchobanou, V. Mironov. Design of multidimensional filter banks. In *Proc. The 2nd International Workshop on Multi-dimensional (nD) Systems NDS-2000*, pages 183–188, Zielona Góra, Poland, 2000.
9. M. Tchobanou, C. Woodburn. Design of M-D filter banks by factorization of M-D polynomial matrices. In *Proc. 3rd International Conference on Information, Communications, and Signal Processing - ICICS'2001*, Singapore, 2001.
10. P. P. Vaidyanathan. *Multirate Systems and Filter Banks*. Prentice Hall, Englewood Cliffs, 1993.

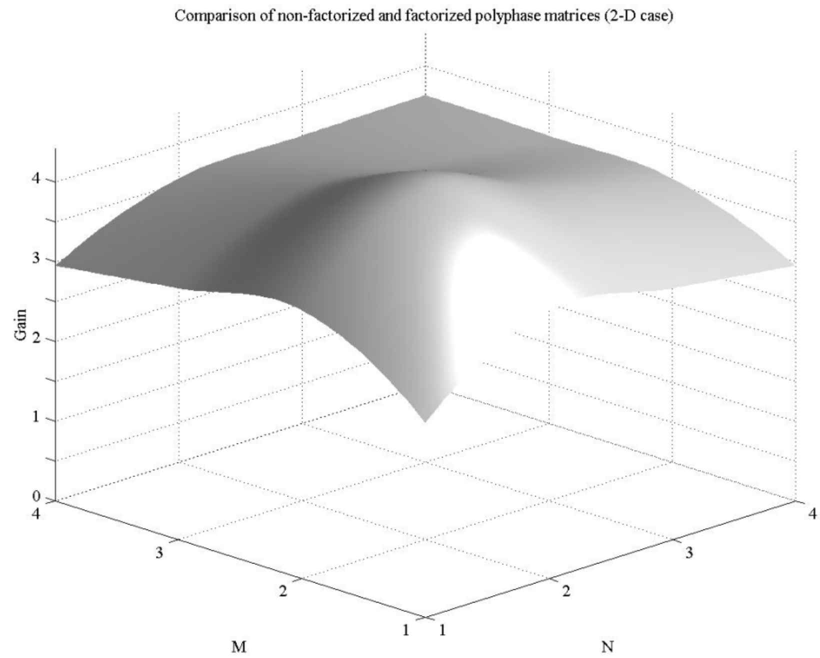

Fig.1 Comparison of non-factorized and factorized polyphase matrices (2-D case)