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FINITE WORDLENGTH COEFFICIENT DESIGN OF DIGITAL FILTERS BASED ON ALLPASS NETWORKS

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Abstract. An algorithm using a variation of initial parameters for finite wordlength coefficient design of digital filters based on parallel connection of two allpass networks is proposed. At the first stage good starting points in an elliptic approximation initial parameter space are defined and on the second a local search of solutions in vicinities of these points is executed. A conventional and two special variants of filter design are considered. Some examples of the minimum coefficient wordlength or/and efficient multiplierless filter design are presented.

1.Introduction

An optimal solution of the design problem of digital filters with finite wordlength coefficients, implemented on custom or semi-custom VLSI, permits to reduce the chip area, power consume, requirements to speed of operation and cost. In order to obtaine these solutions one of the methods using a variation of coefficients (VC) or a variation of initial parameters (VIP) or their combination can be applied [1]. By initial parameters we mean the passband ripple Δa, stopband attenuation a_0 and the edge frequencies of these bands f_j. One of advantages of VIP-method is small dimension of an optimization problem, not dependent on a filter order N. The essence of the method consists in a search of such point in the initial parameter space S for which after computation and quantization of coefficients the filter magnitude response will satisfy to the given tolerance scheme.

An important moment is a choice of good starting points in the space S. In [2] a method of definition of such points is offered. According to this method the parameters in S are chosen so that for a cascade IIR digital filter the coefficients appropriated to the dominant pole-zero pair appear quantized without their priori quantization. The method [2] is used in VIP-algorithm [3]. The efficiency of the algorithm is confirmed on filter design examples with the minimum coefficient wordlength and minimum total number of adders in multiplierless realizations. The possible updatings of the algorithm are presented in [4, 5]. A combination of VIP- and VC-algorithm can result in some improvement of solutions [1].

The authors [6-8] have applied the idea of good starting points to design of digital filters based on parallel connection of two allpass networks with the minimum number of shift-and-add operations in a coefficient representation. Each circuit is realized as a cascade connection of allpass sections not higher second-order with a specific transfer function. The elliptic digital filters appropriated to a minimal Q-factors analog prototype [9] (we shall name their as digital Q_{min}-filters) are chosen. All this has allowed at the first filter design stage to obtain (N+1)/2 dominant quantized coefficients without their priori quantization [8]. At the second stage VC-method for search quantized values of other (N+1)/2 coefficients is applied. The ranges of their possible values and sequence of quantization are defined by the coefficient sensitivities.

In this paper VIP-algorithm for design of digital filters based on parallel connection of two allpass networks with the minimum coefficient wordlength and/or minimum total number of adders in multiplierless realizations is proposed. The algorithm is similar to the one described in [3] and contains two stages. At the first stage good starting points in S are defined and on the second a local search of solutions in vicinities of these points is executed. The variants of definition of starting points for three types of elliptic digital filters are considered: conventional, Q_{min}-filters and half-band filters. On some examples it is shown that the proposed algorithm gives the best results in comparison to obtained in [6,8].

2. Definition of starting points

The idea of the method [2] consists in following. The coefficient vector **C** of IIR digital filter is a vector-function of an approximation initial parameter vector **p**, i.e.

$$\mathbf{C} = \mathbf{F}(\mathbf{p}),\ \ \mathbf{p} \in S(\mathbf{p}),$$

where S(**p**)- the space of all tolerable initial parameters, its sizes are defined by filter specifications and N.

For any point in S the parameters of a filter $\Delta a \le \Delta a_{max}$ and $a_0 \ge a_{0min}$ at $f_1 = f_{1g}$ и $f_2 = f_{2g}$. Here in right parts are given values. The dimension of p depends on the types of approximation (elliptic, Chebyshev et. al.) and the filter (low-pass, band-pass et. al.). The dimension of **C** depends on N. For low- or high-pass elliptic filters

$$\mathbf{p} = (p_1,p_2,p_3) = (\Delta a,f_1,f_2).$$

The starting points inside $S(\Delta a,f_1,f_2)$ are chosen so, that the computation of the vector **C** results to three quantized coefficients without their priori quantization. For cascade filters it is offered to connect these coefficients with the dominant pole-zero pair. The maximal coefficient quantization step q_{max} is chosen so that after the quantization with $q=q_{max}$ at least one of the above points will be inside S, and with $q>q_{max}$ none of them. The S is a comprehensive space of tolerable initial parameters. Therefore for the second-order filters the decision of the system at $q=q_{max}$ immediately results in the global minimum coefficient wordlength $M_{min} = -\log_2(q_{max})$. The graphic interpretation of a starting point is given in [3].
We shall consider now elliptic digital filters based on parallel connection of two cascade allpass circuits containing sections with transfer functions

$$\frac{\alpha_i + z^{-1}}{1+\alpha_i z^{-1}}, \quad i = 1 \quad \text{and} \quad \frac{\beta_i + \alpha_i(1+\beta_i)z^{-1} + z^{-2}}{1+\alpha_i(1+\beta_i)z^{-1} + \beta_i z^{-2}}, \quad i = 2,3,\ldots,K.$$

Here the $K=(N+1)/2$ and N is an odd number. The sections with odd i form the one of two allpass networks and with even i - the other. The enumeration of coefficients α_i and β_i differs from the used in [6-8]. It may be shown that the coefficients are certain functions on the parameters

$$\alpha_1 = \Phi_1(\Delta a,f_1),\ \alpha_i = \Phi_i(\Delta a,f_1,f_2),\ \beta_i = \Psi_i(\Delta a,f_1,f_2),\ i= 2,3,...,K.$$

We shall present variants of elliptic filters: conventional and special filters. Special variants are Q_{min}- and half-band filters. For Q_{min}-filter a value N can be higher than for a conventional filter, and for a half-band filter - higher than for Q_{min}-filter, and the coefficients of special filters do not depend on Δa [6-8].
In the first conventional variant the starting points inside $S(\Delta a,f_1,f_2)$ we shall define by a decision the following system of three equations

$$\alpha_1 = \Phi_1(\Delta a,f_1),\ \alpha_K = \Phi_K(\Delta a,f_1,f_2),\ \beta_K = \Psi_K(\Delta a,f_1,f_2)$$

for sets of quantized coefficient values α_1, α_K, β_K. The coefficients with $i = K$ correspond the dominant pole of the filter transfer function. For the third order filters the decision of the system at $q=q_{max}$ immediately results in the global minimum coefficient wordlength M_{min}.
For Q_{min}-filters the starting points in $S()=S(f_1,f_2)$ we shall define from the following system of two equations

$$\alpha_K = \Phi_K(f_1,f_2),\ \beta_K = \Psi_K(f_1,f_2)$$

for sets of quantized coefficient values α_K, β_K. In the first equation the parameters are dependent and the $\alpha_2() = \alpha_3() = ... = \alpha_K()$ [6-8]. In [6,7] only the first equation was decided, and in [8] - all system.
For half-band digital filters $f_1+f_2 = 0{,}5$, $S() = S(f_1)$ or $S(f_2)$ и $\alpha_i = 0$, $i = 1, 2, \ldots, K$. Here and further the frequencies f_1 and f_2 are normalized in relation to a sampling frequency. The starting points in the $S(f_2)$ we shall define from following equation

$$\beta_K = \Psi_K(f_2)$$

for a set of the quantized coefficient values β_K. All above-stated is fair also and for direct form allpass sections in which coefficients $A_{11} = \alpha_1$, $A_{1i} = \alpha_i(1+\beta_i)$ and $A_{2i} = \beta_i$, $i = 2, 3,.., K$. It is possible to show, that the above systems of equations are reduced to the decision of one equation. For search of starting points we shall allow some expansion S and use a branch and bound technique. Researches of VIP-algorithm [3] show that tolerable solutions with quantized coefficients can be obtained for points outside of S but near to its edges. This fact we take into account also for discussed filters with N>3.

3. Local search and full algorithm

The local search of solutions with finite wordlength coefficients in vicinities of the starting points is executed. For the first, second and third variant of the filter design, respectively three-, two- and one-parametrical search is used. During the variation of chosen parameter the quantization of coefficients is executed and an objective function is evaluated. The minimax criterion is used. A local search strategy and some its modifications were described in details [3]. Two stages of the full algorithm should be repeated for the q equal to q_{max}, $q_{max}/2$ and so on, until the tolerable solution will be found. As a result we shall obtaine the solution with minimum wordlength coefficients. For a multiplierless design [3] it is desirable to minimize the total number of adders replaced multipliers. In this case it is necessary to define all tolerable solutions, obtained for q_0 and $q_0/2$, and to choose from them the minimum adder solution. Here q_0 is a value q, for which the first tolerable solution is found.

4. Design examples

The proposed VIP-algorithm was applied for design of many filters, including all considered in [6-8]. The algorithm gives the best results in comparison with obtained in [6,8] and permits to repeat results [7] without use of the local search (for a filter with N=3 from [7] the coefficient β_2 should be equal 1-1/4, instead of -1/4). Below some design examples are presented.

4.1. Example 1

The low-pass filter specifications are $\Delta a_{max} = 0.5$dB, $a_{0min} = 25$dB, $f_{1g} = 0.15$, $f_{2g} = 0.3$ and N=3.
The application of the initial point method results in three solutions with $M_{min} = 3$. The best of them in relation to the total number of nonzero bits in CSD- representation of coefficients is

$$S(\Delta a, f_1, f_2):\ S(0.039560, 0.138151, 0.296290),\ \alpha_1 = -2^{-2},\ \alpha_2 = -2^{-2} - 2^{-3},\ \beta_2 = 2^{-1}.$$

The multiplication on α_2 is reduced to one adding, and multiplication on other two coefficients - to shift operations. It is interesting, for the filter realized on the allpass direct form sections there is only one solution at $M_{min}=2$ and with coefficients equal to powers of two

$$S(\Delta a, f_1, f_2):\ S(0.059454, 0.150762, 0.313932),\ \ A_{11} = -2^{-2},\ A_{12} = -2^{-1},\ A_{22} = 2^{-1}.$$

Increase M does not give solutions with smaller total number of non-zero bits. Thus, for the considered structures we have obtained the elliptic digital filters with the global minimum of coefficient wordlength and minimum total number of non-zero bits (or adders). Here we specially emphasize the term elliptic, as N=3. For N>3, strictly speaking, elliptic digital filters for discussed structures with all quantized coefficients does not exist.

4.2. Example 2

The low-pass half-band filter specifications [6] are $\Delta a_{max} = 0.2$dB, $a_{0min} = 65$dB and $f_{2g} = 0.2875$.
We have applied the proposed VIP-algorithm to three above variants of the filter design. As the result

var.1: $N = 7,\ M = 6,\ \Sigma_m = 10,\ \Sigma = 28,\ S(\Delta a, f_1, f_2) = S(0.080198, 0.219004, 0.279019),$
$\alpha_1 = -2^{-1} + 2^{-4},\ \alpha_2 = -2^{-1} - 2^{-5},\ \alpha_3 = -2^{-2} - 2^{-6},\ \alpha_4 = -2^{-3} - 2^{-5},$
$\beta_2 = 2^{-2} + 2^{-3} - 2^{-6},\ \beta_3 = 2^{-1} + 2^{-3} + 2^{-5},\ \beta_4 = 1 - 2^{-3} + 2^{-6};$

var.2: $N = 9,\ M = 9,\ \Sigma_m = 8,\ \Sigma = 31,\ S(f_1, f_2) = S(0.177349, 0.285030),$
$\alpha_1 = -2^{-4} + 2^{-9},\ \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = -2^{-3},$
$\beta_2 = 2^{-4} + 2^{-6},\ \beta_3 = 2^{-2} + 2^{-6} + 2^{-7},\ \beta_4 = 2^{-1} + 2^{-5},\ \beta_5 = 2^{-1} + 2^{-2} + 2^{-4} + 2^{-6};$

var.3: $N = 11,\ M = 8,\ \Sigma_m = 7,\ \Sigma = 23,\ S(f_2) = S(0.290396),$
$\beta_2 = 2^{-4},\ \beta_3 = 2^{-2} - 2^{-5} + 2^{-7},\ \beta_4 = 2^{-1} - 2^{-4},\ \beta_5 = 2^{-1} + 2^{-3} + 2^{-5},\ \beta_6 = 1 - 2^{-3} + 2^{-8},$

where Σ_m is the total number of adders replaced multipliers, and Σ is the total number of adders including the structural adders in sections and the one for summation of outputs of two allpass networks. Here and further for the first-order section a structure with two structural adders (see Fig.1 in [6]) and for the second-order section a structure with five adders (see Fig.2 in [6]) were used. The var.3 (half-band filter) has the least values of Σ_m and Σ. This solution was obtained at the first stage of our algorithm and permits to reduce the values Σ_m on 56 % and Σ on 28 % in comparison with a simple rounding of coefficients at $f_2=f_{2g}$ (M=11). For var.2 (Q_{min}-filter) a solution was found at M=8, but with $\Sigma=34$. In Q_{min}-filter from [6] was quantized only a part of coefficients and quantization in the conventional and half-band filters was not considered.

4.3 Example 3

The low-pass filter specifications [6] are $\Delta a_{max} = 0.01$dB, $a_{0min} = 40$dB, $f_{1g} = 0.25$, $f_{2g} = 0.31$ and N=7.
As a result of application of proposed algorithm for two design variants we have

var.1: $M = 5,\ \Sigma_m = 4,\ \Sigma = 20,\ S(\Delta a,f_1,f_2) = S(0.000749, 0.250199, 0.314137),$
$\alpha_1 = 0,\ \alpha_2 = 2^{-4},\ \alpha_3 = 2^{-3},\ \alpha_4 = 2^{-3}+2^{-5},\ \beta_2 = 2^{-3}+2^{-5},\ \beta_3 = 2^{-1},\ \beta_4 = 1-2^{-3}-2^{-5};$

var.2: $M = 7,\ \Sigma_m = 6,\ \Sigma=24,\ S(f_1,f_2) = S(0.24408, 0.294980),$
$\alpha_1 = 2^{-4},\ \alpha_2 = \alpha_3 = \alpha_4 = 2^{-3},\ \beta_2 = 2^{-3}+2^{-4}+2^{-7},\ \beta_3 = 2^{-1}+2^{-5}+2^{-6},\ \beta_4 = 1-2^{-3}-2^{-6}.$

As can be seen, the var.2 (Q_{min}-filter) concedes to the var. 1 (conventional filter) in relation to the values Σ_m and Σ. For var.1 the coefficient $\alpha_1 = 0$ and first-order section is reduced to a delay element. It is interesting, a simple rounding of minimax filter coefficients at $f_1=f_{1g}$, $f_2=f_{2g}$ gives a solution for which Σ_m on 67% and Σ on 33% more than for var.1. In Q_{min}-filter from [6] was quantized only a part of coefficients and quantization in the conventional filter was not considered. For this example many solutions of acceptable on a_0 and not acceptable on Δa and on the contrary is met. It justifies application of a minimax criterion in the proposed algorithm, instead of the control only a_0, as in [8].

4.4. Example 4

The low-pass filter specifications [8] are $\Delta a_{max} = 0.2$dB, $a_{0min} = 30$dB, $f_{1g} = 0.135$, $f_{2g} = 0.2$ and N=5.
As a result of application of proposed algorithm for two design variants we have

var.1: $M = 4,\ \Sigma_m = 3,\ S(\Delta a,f_1,f_2) = S(0.000202, 0.105778, 0.217013),$
$\alpha_1 = -2^{-2},\ \alpha_2 = \alpha_3 = -2^{-1}-2^{-4},\ \beta_2 = 2^{-2},\ \beta_3 = 2^{-1}+2^{-2}.$

var.2: $M = 4,\ \Sigma_m = 5,\ S(f_1,f_2) = S(0.114473, 0.203756),$
$\alpha_1 = -2^{-2}-2^{-4},\ \alpha_2 = \alpha_3 = -2^{-1}-2^{-4},\ \beta_2 = 2^{-2}+2^{-4},\ \beta_3 = 2^{-1}+2^{-2}.$

The solution from [8] is

$$M = 7,\ \Sigma_m = 6,\ \alpha_1 = -2^{-2}-2^{-6},\ \alpha_2 = \alpha_3 = -2^{-1},\ \beta_2 = 2^{-1}-2^{-3}-2^{-5},\ \beta_3 = 1-2^{-2}+2^{-5}+2^{-7}.$$

As can be seen, the proposed algorithm results in improvement of the solution [8] in relation to Σ_m. For var.1 the solution corresponds to conventional design, but presents Q_{min}-filter. For this example it is impossible to obtain this solution purposely using Q_{min}-filter design. The initial point for var.1 was outside of S, but near of its bound. Notice, that the value $\beta_3 = 0.75$ is located outside of a range determined in [8] for this coefficient.

4.5. Example 5

The low-pass half-band filter specifications [8] are $a_{0min} = 46$dB, $f_{2g} = 0.28$ and N=9. The parameter Δa_{max} is not given.
All three design variants result in identical solution

var.1: $S(\Delta a,f_1,f_2) = S(0.000007, 0.219030, 0.280971),$
var.2: $S(f_1,f_2) = S(0.219030, 0.280971),$
var.3: $S(f_2) = S(0.280971),$
$M = 5,\ \Sigma_m = 5,\ \beta_2 = 2^{-4}+2^{-5},\ \beta_3 = 2^{-2}+2^{-4}+2^{-5},\ \beta_4 = 2^{-1}+2^{-3},\ \beta_5 = 1-2^{-3}.$

It is interesting, for all three variants this solution is obtained at the first stage of the proposed algorithm.

The solution from [8] is

$$M = 8,\ \Sigma_m = 7,\ \beta_2 = 2^{-3} - 2^{-8},\ \beta_3 = 2^{-2} + 2^{-3} + 2^{-6},\ \beta_4 = (1 + 2^{-2})(2^{-1} + 2^{-5}),\ \beta_5 = 1 - 2^{-3} + 2^{-6}.$$

As can be seen, the proposed algorithm results in reduction of the value Σ_m on two adders.

5. Conclusions

In this paper a method of variation of elliptic approximation initial parameters is extend on the finite wordlength coefficient design of digital filters based on parallel connection of two allpass networks. The algorithm directed on obtaining of minimum coefficient wordlength filters or/and multiplierless filters with a minimum total number of adders is proposed. At the first stage good starting points in an initial parameter space are defined and on the second a local search of solutions with quantized coefficients in vicinities of these points is executed. Design properties of a conventional and two special filter (Q_{min}-filters and half-band) are considered. The solutions with the minimum total number of adders not always correspond to Q_{min}-filters. Besides the conventional design can result to the best Q_{min}-filter than the direct design of such filter. At half-band requirements, as appear, it should design just half-band filters. It is proved that for considered structures there are conventional third order elliptic digital filters with all quantized coefficients. In this case the initial point method directly result to the global optimal solutions. The proposed algorithm permits to improve the existing solutions in relation to the total number of adders and can be used at development of VLSI multiplierless digital filters and CAD tools for their design.

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