

Michael Z. Lowenstein. "Power Factor Measurement."

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Power Factor Measurement

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Electricity can be thought of as a means of delivering power from one place to another to do work. The laws and relationships for delivering power were originally developed for direct current. Power delivered, expressed in *watts*, was calculated by multiplying the voltage and current as shown in Equation 40.1

$$P_{dc} = EI \quad (40.1)$$

The situation becomes more complex when alternating current is used to deliver power. [Figure 40.1](#) shows a sine wave representing either ac current or voltage. Since the instantaneous value of the wave is continually changing, a numerical quantity is defined to represent an average property of this wave. This quantity, the root-mean-square, or **rms** value, calculated by squaring the instantaneous value, integrating it during one cycle, dividing by the period, and taking the square root of the result, is equal to the peak value of the ac wave divided by the square root of 2, or, for ac current, $I_{rms} = i_{peak}/\sqrt{2}$. In the physical world, a sine wave ac current having an rms value of 1 A (A = ampere), passed through a resistive load, produces the same heating effect as 1 A of dc current. Thus, one might expect delivered ac power to be easily calculated in watts using Equation 40.1 and inserting rms values for current and voltage. While this simple relationship holds true for the instantaneous voltage and current values as shown in Equation 40.1a in general, it is not true for the rms quantities except for the special case when the ac current and voltage are restricted to perfect sine waves *and* the load is a pure resistance.

$$p_{inst.} = ei \quad (40.1a)$$

In real-world situations where current and/or voltage waveforms are not perfectly sinusoidal and/or the loads are other than resistive, the relationships are no longer simple and the power delivered, or *active power*, is usually less than the product of rms voltage and current, as shown in Equation 40.2.

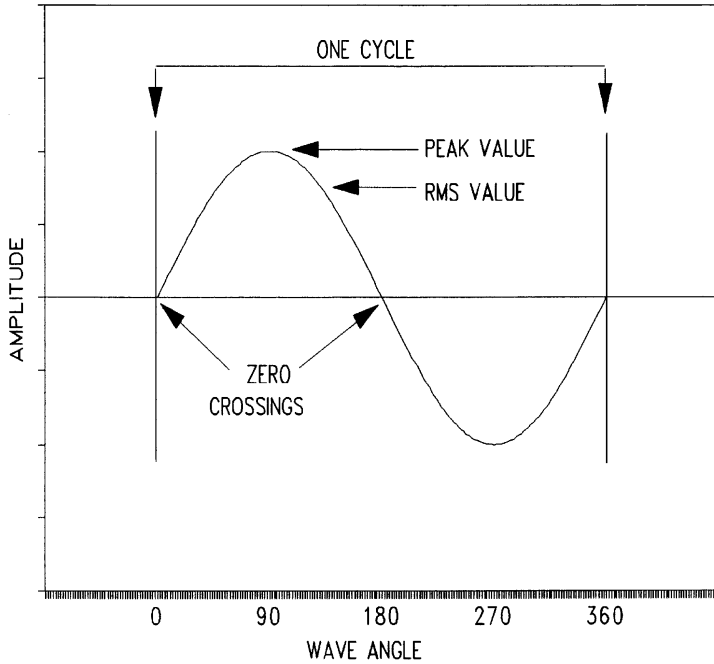


FIGURE 40.1 Sine wave characteristics. One cycle of a continuous wave is shown. The wave begins at a zero-crossing, reaches a positive peak, continues through zero to a negative peak, and back to zero. The wave repeats every 360° . The wave angle can also be expressed as a function of frequency f and time t . For a given frequency, the wave angle is related to the expression, $2\pi ft$.

$$P_{ac} \leq E_{rms} I_{rms} \quad (40.2)$$

The product of rms voltage and rms current does, however, define a quantity termed *apparent power*, U , as shown in Equation 40.3.

$$U = E_{rms} I_{rms} \quad (40.3)$$

A derived term, the *power factor*, F_p , used to express the relationship between delivered or active power, P , and apparent power, U , is defined by Equation 40.4.

$$F_p = \frac{P}{U} \quad (40.4)$$

From Equations 40.2, 40.3, and 40.4, it is clear that the value of F_p must lie in a range between zero and one.

This chapter focuses on: (1) ac power relationships and the calculation of power factor; (2) the physical meaning of these relationships; and (3) measurement techniques and instrumentation for determining these relationships and calculating power factor.

40.1 Reasons for Interest in Power Factor

Power factor is a single number that relates the active power, P , to the apparent power, U . Electric components of a utility distribution system are designed on a kVA basis; i.e., they are designed to operate

at a given voltage and carry a rated current without undue temperature rise. Transformer and conductor sizes are chosen on this basis. While active power does useful work, reactive and **harmonic powers** do no useful work, absorb system capacity, and increase system losses; but reactive and harmonic powers are needed to provide magnetic fields or nonlinear currents. *The capacity of electric systems is limited by apparent power, not active power. Power factor expresses, with a single value, the extent to which an electrical distribution system is efficiently and effectively utilized.* A low value for the power factor means that much of the system capacity is not available for useful work. From a utility viewpoint, this means reduced ability to deliver revenue-producing active power; from a user viewpoint, a low power factor reduces the available active power or requires increased system size to deliver needed power.

40.2 ac Electric Loads

Linear Loads

Electric loads in ac power systems with sinusoidal voltages are categorized by the way they draw current from the system. Loads that draw sinusoidal currents, i.e., the current waveshape is the same as the voltage waveshape, are defined as *linear loads*. Historically, a high percentage of electric loads have been linear. Linear loads include: (1) induction motors; (2) incandescent lighting; and (3) heaters and heating elements. Linear loads use ac electric power directly to accomplish their functions.

Nonlinear Loads

Electric loads that draw nonsinusoidal currents, i.e., the current waveshape differs from the voltage waveshape, are defined as *nonlinear loads*. As energy savings and efficient use of electricity are emphasized, an increased percentage of nonlinear electric devices, both new and replacement, are being installed. Nonlinear loads include: (1) adjustable-speed motor drives; (2) fluorescent and arc-discharge lighting; (3) computers and computerized controls; and (4) temperature-controlled furnaces and heating elements. Nonlinear loads, rather than using ac electric power directly, often convert ac power into direct current before it is used to accomplish their functions. A common element in nonlinear loads is some kind of rectifier to accomplish this ac to dc conversion. Rectifiers do not draw sinusoidal currents.

40.3 ac Power Relationships

Sinusoidal Voltage and Current

Power calculations for sinusoidal ac electric systems require knowledge of the rms voltage, the rms current, and the phase relationships between the two. **Figure 40.2** illustrates possible phase relationships between voltage and current. If the positive-going zero-crossing of the voltage is considered the reference point, then the nearest positive-going zero-crossing of the current can occur at a wave angle either less than or greater than this reference. If the current zero-crossing occurs before the reference, the current is said to *lead* the voltage. If the current zero-crossing occurs after the reference, the current is *lagging*. If the zero-crossing for the current coincides with the reference, the two waves are said to be *in phase*. The wave angle, θ , by which the current leads or lags the voltage is called the *phase angle*, in this case, 30° .

Single-Phase Circuits

Power Calculations

The power delivered to do work is easily calculated [1]. Given a sinusoidal voltage of rms magnitude E and sinusoidal current of rms magnitude I , displaced by angle θ , at time t ,

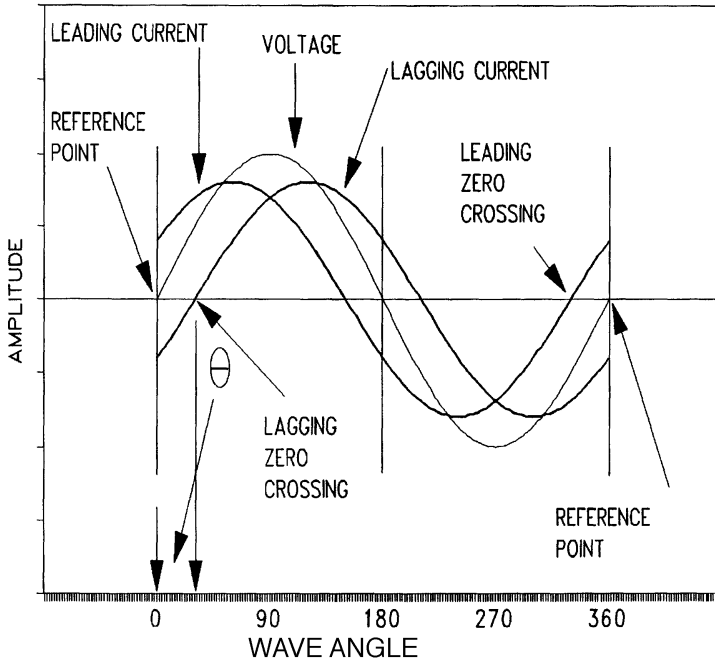


FIGURE 40.2 Sine wave phase angle. Two waves with the same zero-crossing are *in phase*. A sine wave that crosses zero before the reference wave is *leading*, and one that crosses zero after the reference wave is *lagging*. The *phase angle* θ illustrated is 30° lagging.

$$\text{Instantaneous voltage} = e = \sqrt{2} E \sin(2\pi ft)$$

$$\text{Instantaneous current} = i = \sqrt{2} I \sin(2\pi ft - \theta) \quad (\text{note that } \theta \text{ can have a positive or negative value}) \quad (40.5)$$

$$\text{Instantaneous power} = p = ei$$

$$p = 2EI \sin(2\pi ft) \sin(2\pi ft - \theta)$$

$$p = EI \cos(\theta) - EI \cos(4\pi ft - \theta)$$

$$\text{Average power over an integral number of cycles} \quad P = EI \cos(\theta) \quad (40.5)$$

$$\text{Power factor } F_p = \frac{P}{U} = \frac{EI \cos(\theta)}{EI} = \cos(\theta) \quad (40.5)$$

Equation 40.5 is the fundamental equation that defines power for systems in which the current and voltage are sinusoidal. The application of this equation is illustrated for three cases: (1) the current and voltage are in phase; (2) the current and voltage are out of phase by an angle less than 90° ; and (3) the current and voltage are out of phase by exactly 90° .

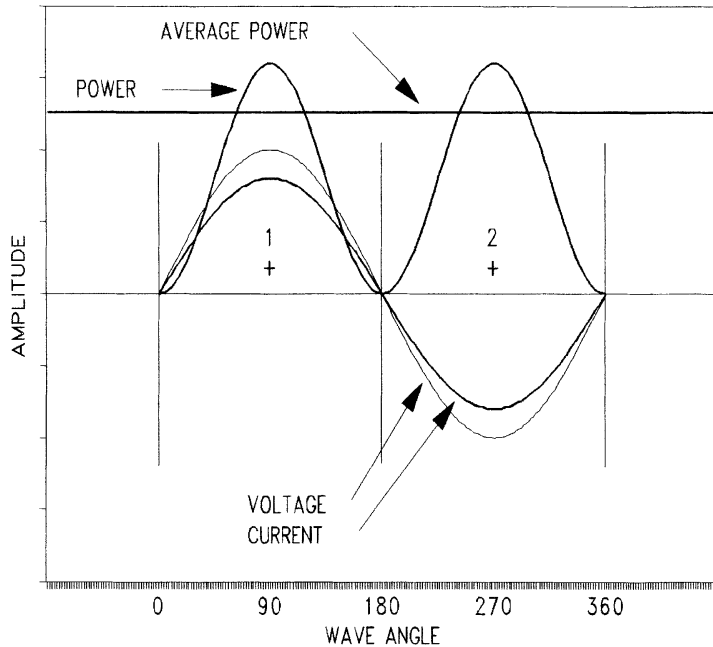


FIGURE 40.3 Voltage, current, and power for sine waves in phase. The vertical scales for voltage and current are equal. The scale for power is relative and is selected to permit the display of all three waves on a single graph. The voltage and current are in phase and both are sinusoidal. The power is everywhere positive, and average power delivered to do work is the maximum power.

Ac Power Examples

Figure 40.3 shows voltage current and power when the voltage and current are in phase and the current displacement angle is zero (0°). (An example would be a resistance space heater.) The power curve is obtained by multiplying together the instantaneous values of voltage and current as the wave angle is varied from 0° to 360° . Instantaneous power, the product of two sine waves, is also a sine wave. There are two zero-crossings per cycle, dividing the cycle into two regions. In region (1) both the voltage and current are positive and the resultant product, the power, is positive. In region (2) both the voltage and current are negative and the power is again positive. The average power in watts, given by Equation 40.5, $EI\cos(0^\circ) = EI$, is the maximum power that can be delivered to do work. *When sinusoidal voltage and current are in phase, the delivered power in watts is the same as for dc and is the maximum that can be delivered.* The power factor, $F_p = \cos(0^\circ) = 1$, or *unity*.

Figure 40.4 shows voltage, current, and power when the current lags the voltage by 60° . (An example might be a lightly loaded induction motor.) The power sine wave again is obtained by multiplying together the instantaneous values of voltage and current. There are now four zero-crossings per cycle, dividing the cycle into four regions. In regions (2) and (4), voltage and current have the same sign and the power is positive. In regions (1) and (3), voltage and current have opposite signs, resulting in a negative value for the power. The average power in watts, given by Equation 40.5, $EI\cos(60^\circ) = EI(0.5)$, is less than the maximum that could be delivered for the particular values of voltage and current. *When voltage and current are out of phase, the delivered power in watts is always less than the maximum.* In this example, $F_p = \cos(60^\circ) = 0.5$.

Figure 40.5 shows voltage, current, and power when the current lags the voltage by 90° . (This situation is not attainable in the real world.) The power sine wave again is obtained by multiplying together the instantaneous values of voltage and current. Again, four zero-crossings divide the cycle into four regions.

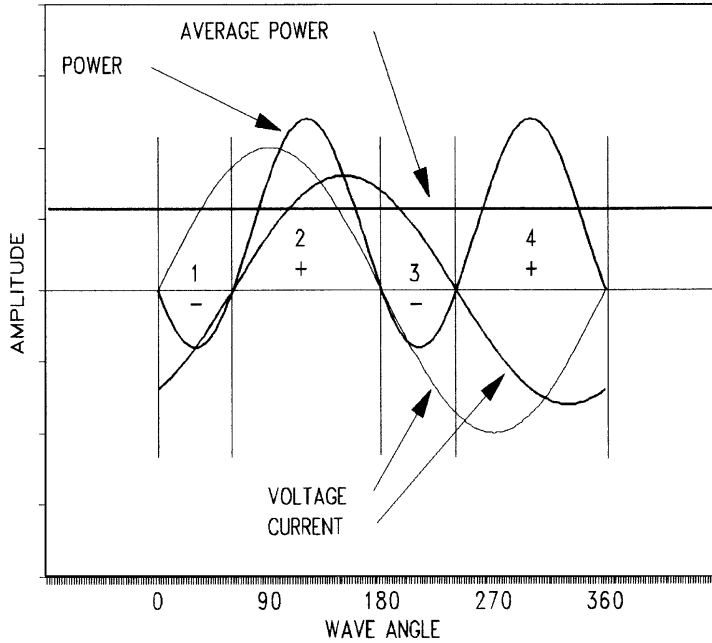


FIGURE 40.4 Voltage, current, and power for sine waves 60° out of phase. The vertical scales for voltage and current amplitudes are the same as those for Figure 40.3. Current lags voltage by 60° and both are sinusoidal. The power is positive in regions (2) and (4), and negative in regions (1) and (3). Average power delivered to do work is less than the maximum power.

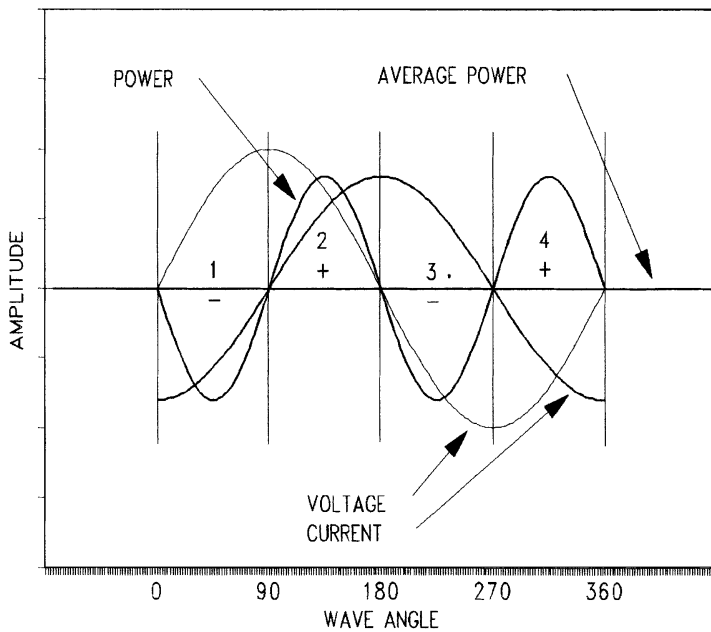


FIGURE 40.5 Voltage, current, and power for sine waves 90° out of phase. The vertical scales for voltage and current amplitudes are the same as those for figure 40.3. Current lags voltage by 90° and both are sinusoidal. The power is positive in regions (2) and (4), negative in regions (1) and (3), and is of equal absolute magnitude in all four regions. Average power delivered to do work is zero.

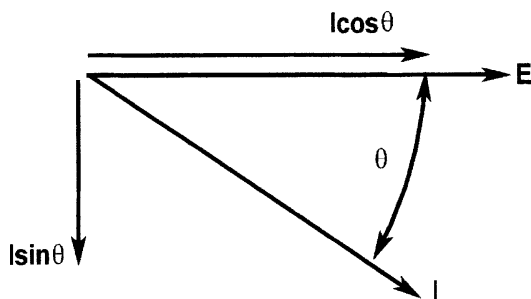


FIGURE 40.6 Phasor diagram for current and voltage. Voltage is the reference phasor. The current I has been resolved into orthogonal components $I\cos\theta$ and $I\sin\theta$. (From Reference [1].)

In regions (2) and (4), the power is positive, while in regions (1) and (3), the power is negative. The average power in watts is given by Equation 40.5, $EI\cos(90^\circ) = 0$. No matter what the values of voltage and current, *when voltage and current are exactly 90° out of phase, the delivered power in watts is always zero.* The power factor, $F_p = \cos(90^\circ) = \text{zero}$.

Power Factor

Resolving the current into orthogonal components on a phasor diagram illustrates how delivered power can vary from a maximum to zero, depending on the phase angle between the voltage and the current sine waves. Figure 40.6 shows the voltage vector along with the current resolved into orthogonal components [1]. The current I at an angle θ relative to the voltage can be resolved into two vectors: $I\cos(\theta)$ and $I\sin(\theta)$. The in-phase component $I\cos(\theta)$ multiplied by the voltage gives average power in watts. The current component that is 90° out of phase with the voltage, $I\sin(\theta)$, is not associated with delivered power and does not contribute to work. For want of a better name, this was often termed the *wattless component* of the current. Since this wattless current could be associated with magnetic fields, it was sometimes termed *magnetizing current* because, while doing no work, this current interacts through the inductive reactance of an ac motor winding to provide the magnetic field required for such a motor to operate.

Three types of power have been defined for systems in which both the voltage and current are sinusoidal. Throughout the years, a number of different names have been given to the three power types. The names in present usage will be emphasized.

Active power is given the symbol P and is defined by Equation 40.5:

$$P = EI\cos(\theta) \quad (40.5)$$

Other names for active power include: (1) real power and (2) delivered power. Active power is the power that does work. Note that while all power quantities are volt-ampere products, only *active power* is expressed in watts.

Reactive power is given the symbol Q and is defined by the equation:

$$Q = EI\sin(\theta) \quad (40.6)$$

Other names for reactive power include: (1) imaginary power; (2) wattless power; (3) and magnetizing power. Reactive power is expressed in *voltamperes_{reactive}* or *vars*. If the load is predominantly inductive, current lags the voltage and the reactive power is given a positive sign. If the load is predominantly capacitive, current leads the voltage and the reactive power is given a negative sign.

Phasor power is given the symbol S and is defined by the equation:

$$S = \sqrt{P^2 + Q^2} \quad (40.7)$$

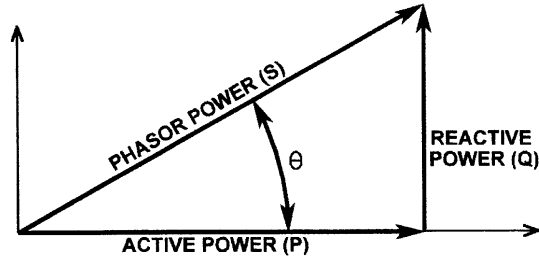


FIGURE 40.7 Power triangle showing the geometric relationships between active, reactive, and phasor power. Power factor is defined geometrically as $\cos\theta$.

Phasor power was called apparent power for many years, and it will be seen in a later section that phasor power S , for sinusoidal voltages and currents, is identical to what is now called apparent power U . Phasor power is expressed in *voltamperes* or VA.

Figure 40.7 is a phasor diagram, often called a *power triangle*, which illustrates the relationships among the three types of power defined above. Reactive power is orthogonal to active power, and is shown as positive for lagging current. It is clear that the definition of phasor power, Equation 40.7, is geometrically derived from active and reactive power.

Power factor is given the symbol F_p and for sinusoidal quantities is defined by the equation:

$$F_p = \frac{\text{ACTIVE POWER}}{\text{PHASOR POWER}} = \frac{P}{S} = \frac{\text{WATTS}}{\text{VOLTAMPS}} = \cos\theta \quad (40.8)$$

Since the power factor can be expressed in reference to the displacement angle between voltage and current, power factor so defined should be termed *displacement power factor*, and the symbol is often written $F_{p \text{ displacement}}$. Values for displacement power factor range from one (unity) to zero as the current displacement angle varies from 0° (current and voltage in phase) to 90° . Since the cosine function is positive in both the first and fourth quadrants, the power factor is positive for both leading and lagging currents. To completely specify the voltage–current phase relationship, the words *leading* or *lagging* must be used in conjunction with power factor. Power factor can be expressed as a decimal fraction or as percent. For example, the power factor of the case shown in Figure 40.4 is expressed either as 0.5 lagging or 50% lagging.

Polyphase Circuits

Power Calculations

The power concepts developed for single-phase circuits with sinusoidal voltages and currents can be extended to polyphase circuits. Such circuits can be considered to be divided into a group of two-wire sets, with the neutral conductor (or a resistively derived neutral for the case of a delta-connected, three-wire circuit) paired with each other conductor. Equations 40.3–40.5 can be rewritten to define power terms equivalent to the single-phase terms. In these equations, k represents a phase number, m is the total number of phases, and α and β are, respectively, the voltage and current phase angles with respect to a common reference frame.

$$P = \sum_{k=1}^m E_k I_k \cos(\alpha - \beta) \quad (40.9)$$

$$Q = \sum_{k=1}^m E_k I_k \sin(\alpha - \beta) \quad (40.10)$$

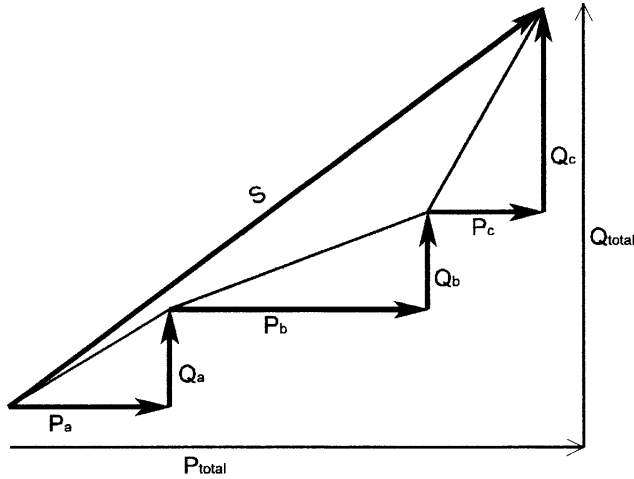


FIGURE 40.8 Phasor diagram for a sample polyphase sinusoidal service in which each phase has a different phase angle. Power factor cannot be defined as the cosine of the phase angle in this case. (From Reference [3].)

and, restating Equation 40.7:

$$S = \sqrt{P^2 + Q^2}$$

For example, a three-phase sinusoidal power distribution service, with phases a, b, and c:

$$P = E_a I_a \cos(\alpha_a - \beta_a) + E_b I_b \cos(\alpha_b - \beta_b) + E_c I_c \cos(\alpha_c - \beta_c)$$

$$Q = E_a I_a \sin(\alpha_a - \beta_a) + E_b I_b \sin(\alpha_b - \beta_b) + E_c I_c \sin(\alpha_c - \beta_c)$$

$$S = \sqrt{P^2 + Q^2}$$

Power Factor

Power factor is defined by Equation 40.11. Note that it is no longer always true to say that power factor is equal to the cosine of the phase angle. In many three-phase balanced systems, the phase angles of all three phases are equal and the cosine relationship holds. In unbalanced systems, such as that represented by the phasor diagram [Figure 40.8](#), each phase has a different phase angle, the phase voltages and currents are not equal, and the cosine relationship fails [3].

$$F_p = \frac{\text{TOTAL ACTIVE POWER}}{\text{PHASOR POWER}} = \frac{P_{\text{Eq.9}}}{S} = \frac{\text{WATTS}}{\text{VOLTAMPS}} \text{ often } \neq \cos\theta \quad (40.11)$$

Nonsinusoidal Voltage and Current

Fourier Analysis

[Figure 40.9](#) shows voltage, current, and power for a typical single-phase nonlinear load, a computer switch-mode power supply. Due to the nature of the bridge rectifier circuit in this power supply, current is drawn from the line in sharp spikes. The current peak is only slightly displaced from the voltage peak and the power is everywhere positive. However, power is delivered to the load during only part of the cycle and the average power is much lower than if the current had been sinusoidal. The current waveshape

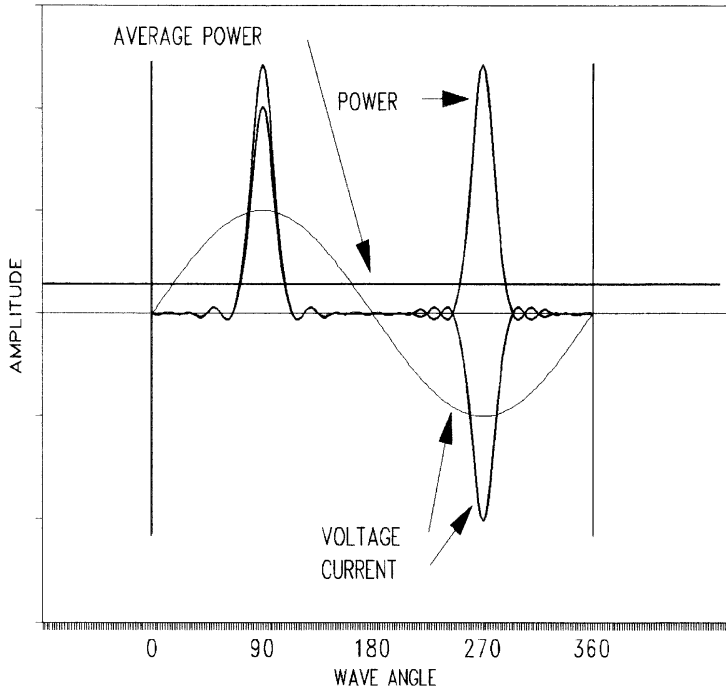


FIGURE 40.9 Voltage, current, and power for a single-phase, switch-mode computer power supply, a typical nonlinear load. The current is no longer sinusoidal.

required by the load presents a problem to the ac power system, which is designed to deliver only sine wave current. The solution to this problem is based on mathematical concepts developed in 1807 for describing heat flow by Jean Baptiste Joseph Fourier, a French mathematician [4]. Fourier's theorem states that any periodic function, however complex, can be broken up into a series of simple sinusoids, the sum of which will be the original complex periodic variation. Applied to the present electrical problem, Fourier's theorem can be stated: *any periodic nonsinusoidal electrical waveform can be broken up into a series of sinusoidal waveforms, each a harmonic of the fundamental, the sum of which will be the original nonsinusoidal waveform.*

Harmonics

Harmonics are defined as continuous integral multiples of the fundamental waveform. Figure 40.10 shows a fundamental sine wave and two harmonic waves — the 3rd and 5th harmonics. The harmonic numbers 3 and 5 express the number of complete cycles for each harmonic wave per cycle of the fundamental (or 1st harmonic). Each harmonic wave is defined by its harmonic number, its amplitude, and its phase relationship to the fundamental. Note that the fundamental frequency can have any value without changing the harmonic relationships, as shown in Table 40.1.

Power Calculations

Calculating power delivered to do work for a nonlinear load is somewhat more complicated than if the current were sinusoidal. If the fundamental component of the voltage at frequency f is taken as a reference (the a-phase fundamental for a polyphase system), the subscript "1" means the fundamental, and E denotes the peak value of the voltage; then the voltage can be expressed as:

$$\varepsilon_{a1}(t) = E_{a1} \sin(2\pi ft + 0^\circ)$$

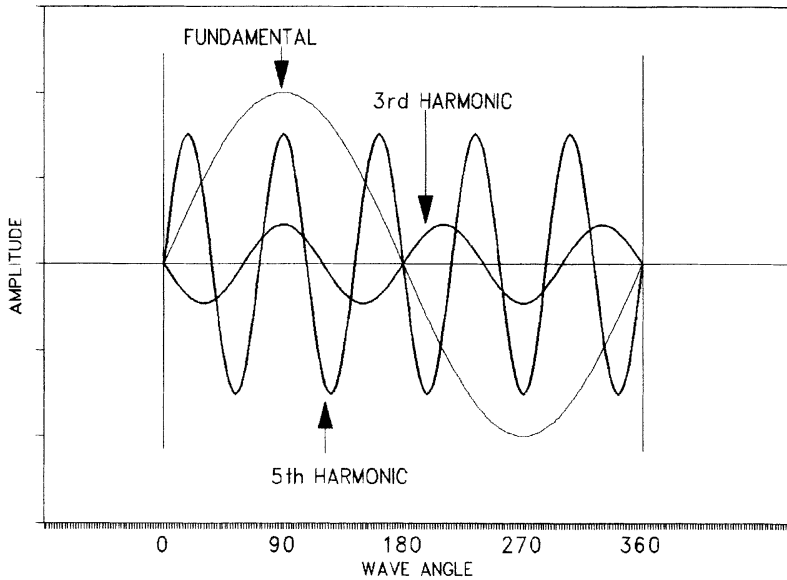


FIGURE 40.10 Harmonics are continuous integral multiples of the fundamental frequency. The 5th harmonic is shown, in phase with the fundamental, while the 3rd harmonic is 180° out of phase.

TABLE 40.1 Harmonics and Their Relationship to the Fundamental Frequency

Harmonic number	Frequency (Hz)	Frequency (Hz)	Frequency (Hz)
1	60	50	400
2	120	100	800
3	180	150	1200
4	240	200	1600
5	300	250	2000

The voltage fundamental will then have an amplitude E_{a1} and pass through zero in the positive going direction at time $t = 0$. If $h =$ harmonic number, and E_h and I_h are peak amplitudes of the harmonic voltage and current, respectively, then general expressions for any harmonic will be:

$$e_{h(t)} = E_h \sin(2\pi fht + \alpha_h^\circ)$$

$$i_{h(t)} = I_h \sin(2\pi fht + \beta_h^\circ)$$

To compute the power associated with a voltage and current waveform, take advantage of the fact that products of harmonic voltages and currents of different frequency have a time average of zero. Only products of voltages and currents of the same frequency are of interest, giving a general expression for harmonic power as:

$$p_{h(t)} = E_h I_h \sin(2\pi fht + \alpha_h^\circ) \sin(2\pi fht + \beta_h^\circ)$$

Simplifying with trigonometric identities, evaluating over an integral number of cycles, and replacing peak voltage and current with rms values, the average power becomes:

$$P_{h(t)} = E_h I_h \cos(\alpha_h^\circ - \beta_h^\circ)$$

For a *single-phase system* where h is the harmonic number and H is the highest harmonic, the total average power or active power is given by:

$$P = \sum_{h=1}^H E_h I_h \cos(\alpha_h - \beta_h) \quad (40.12)$$

Total average reactive power is given by:

$$Q = \sum_{h=1}^H E_h I_h \sin(\alpha_h - \beta_h) \quad (40.13)$$

It should be noted that in the real world, the actual contribution of harmonic frequencies to active and reactive power is small (usually less than 3% of the total active or reactive power). The major contribution of harmonic frequencies to the power mix comes as distortion power, which will be defined later.

For a *polyphase system* wherein r is the phase identification and N is the number of conductors in the system, including the neutral conductor, total average power for a polyphase system is given by:

$$P = \sum_{r=1}^{N-1} \sum_{h=1}^H E_{rh} I_{rh} \cos(\alpha_{rh} - \beta_{rh}) \quad (40.14)$$

Total average reactive power is given by:

$$Q = \sum_{r=1}^{N-1} \sum_{h=1}^H E_{rh} I_{rh} \sin(\alpha_{rh} - \beta_{rh}) \quad (40.15)$$

Power Factor

Single-Phase Systems

For a single-phase system, phasor power is again given by Equation 40.7 and illustrated by Figure 40.7, where P is the algebraic sum of the active powers for the fundamental and all the harmonics (Equation 40.12), and Q is the algebraic sum of the reactive powers for the fundamental and all the harmonics (Equation 40.13). Therefore, phasor power is based on the fundamental and harmonic active and reactive powers. It is found, however, that phasor power S is no longer equal to apparent power U and a new power phasor must be defined to recognize the effects of waveform distortion. A phasor representing the distortion, termed *distortion power* and given the symbol D , is defined by:

$$D = \pm \sqrt{U^2 - S^2} \quad (40.16)$$

Without further definite information as to the sign of distortion power, its sign is selected the same as the sign of the total active power. The relationships among the various power terms are displayed in

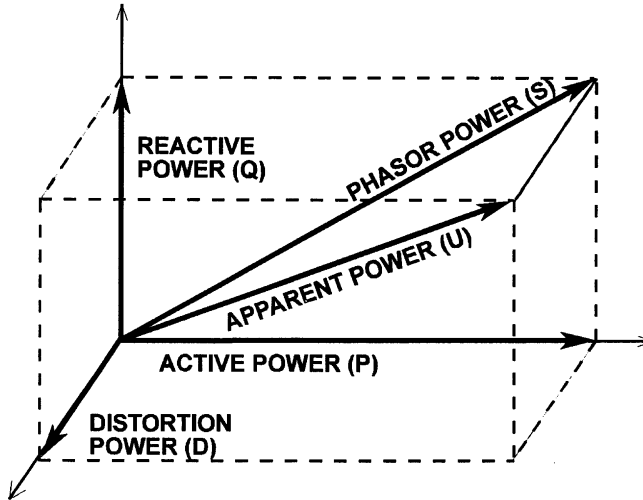


FIGURE 40.11 Phasor diagram for a single-phase, nonsinusoidal service in which the voltage and current contain harmonics. Geometric relationships are shown between active, reactive, phasor, distortion, and apparent powers.

Figure 40.11, a 3-dimensional phasor diagram. Power factor, in direct parallel with sinusoidal waveforms, is defined by the equation:

$$F_p = \frac{\text{TOTAL ACTIVE POWER}}{\text{APPARENT POWR}} = \frac{P}{U} = \frac{\text{WATTS}}{\text{VOLTAMPS}} \quad (40.17)$$

From Equations 40.7 and 40.16 we obtain:

$$S = \sqrt{P^2 + Q^2} \quad (40.7)$$

$$U = \sqrt{S^2 + D^2} = \sqrt{P^2 + Q^2 + D^2} \quad (40.18)$$

It is clear that when waveforms are sinusoidal, i.e., linear loads are drawing current, that there is no distortion power and Equation 40.18 reduces to Equation 40.7. Likewise as shown in Figure 40.13, as the distortion power vector goes to zero, the figure becomes two-dimensional and reduces to Figure 40.7, and U becomes equal to S . When, however, nonlinear loads are drawing harmonic currents from the system, U will be greater than S . As already noted, the contribution of the harmonics to the total power quantities is small and one is frequently interested mainly in the fundamental quantities.

The power factor associated only with the fundamental voltage and current components was termed the displacement power factor $F_{p \text{ displacement}}$ where Equations 40.7 and 40.8 are written [5]:

$$S_{60} = \sqrt{P_{60}^2 + Q_{60}^2}$$

and

$$F_{p \text{ displacement}} = \frac{P_{60}}{S_{60}}$$

When harmonics are present, F_p is always smaller than $F_{p \text{ displacement}}$.

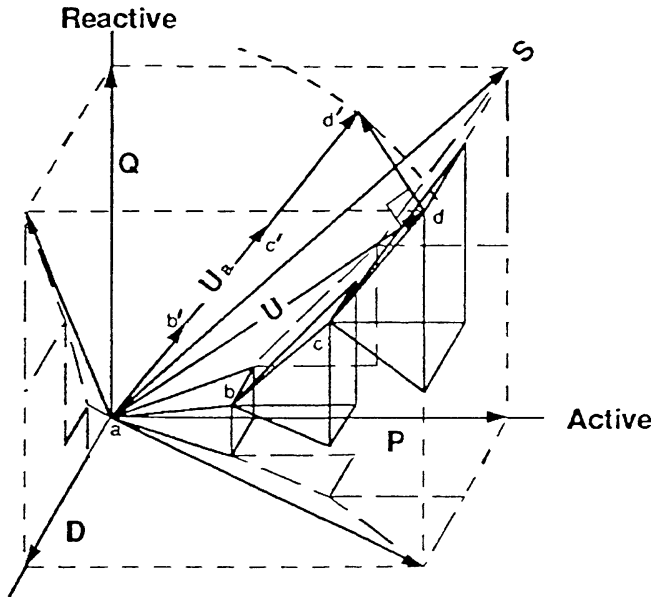


FIGURE 40.12 Phasor diagram for a three-phase, nonsinusoidal service in which the voltage and current contain harmonics. Arithmetic apparent power U_a is the length of the segmented line $abcd$ and is a scalar quantity U_a can be represented by the line $ab'c'd'$. The diagonal ad , a vector quantity, is the apparent power U [6].

Polyphase Systems

For a polyphase system, phasor power, S , is again given by Equation 40.7, but one must now use the total values for P and Q calculated using Equations 40.14 and 40.15. One can then define the apparent power U in one of two ways.

- *Arithmetic apparent power.* The *arithmetic apparent power* is given the symbol U_a , and is defined by Equation 40.19, where E_r and I_r are the rms values for the respective phases and M equals the number of phases. U_a is a scalar quantity.

$$U_A = \sum_{r=1}^{M-1} E_r I_r \quad (40.19)$$

- *Apparent power.* *Apparent power* is given the symbol U and is defined by Equation 40.18 using total values for P and Q as defined by Equations 40.14 and 40.15, and a total value for D determined using Equation 40.16 for each phase. Figure 40.12 illustrates the two alternative concepts for polyphase apparent power [6]. Note that U_a uses arithmetic addition of vector magnitudes and is equal to apparent power U only if the polyphase voltages and currents have equal magnitudes and equal angular spacings, a situation that often exists in balanced three-phase systems. The two alternative definitions of apparent power, U and U_a , give rise to two possible values for power factor: (1) $F_p = P/U$; and (2) $F_{pa} = P/U_a$. Apparent power U and power factor F_p are the preferred definitions since using U_a can give unexpected results with some nonsymmetric service arrangements such as four-wire delta, and, with extremely unbalanced resistive loads, F_{pa} can exceed 1.0. Despite these shortcomings, arithmetic apparent power has become quite widely used due to the comparative simplicity of its measurement and calculation. With the advent of sophisticated digital meters, there is no longer any advantage to using arithmetic apparent power and its use will surely decrease.

40.4 Power Factor “Measurement”

There are no instruments that measure power factor directly. (Power stations and large substations often use phase angle meters with a power factor scale representing $\cos(\theta)$ to display power factor. Such meters are accurate only for sinusoidal balanced polyphase systems.) One must remember that, of all the ac power quantities discussed, the only ones that can be directly measured are voltages, currents, and their time relationships (phase angles). All other ac power quantities are derived mathematically from these measured quantities. The only one of these derived values that has physical reality is the active power P (the quantity that does work); the others are mathematical constructs. Therefore, correct determination of power factor requires accurate measurement of voltage and current, and proficient mathematics.

Metering for Linear Loads

By the early 1920s, the concepts of active, reactive, and apparent (since renamed phasor) power, and power factor were known, and metering capabilities had been developed to enable their determination. Energy meters utilizing voltage and current coils driving an induction disk inherently measured active power P ($EI \cos\theta$), which was displayed on a mechanical register. Using the trigonometric identity $EI \sin\theta = EI \cos(90^\circ + \theta)$, with voltage delayed 90° , a similar energy meter displayed reactive power Q and, with these two values, displacement power factor was calculated. Voltage delay (phase shifting) was accomplished using specially wound transformers.

Through the years, the method of obtaining the 90° phase shift has been updated. Analog electronic meters are available that provide the 90° phase shift electronically within the meter. More recently, digital meters have been developed that sample voltages and currents at regular intervals and digitize the results. Voltages and currents are multiplied as they are captured to compute active power. Past voltage samples delayed by a time equal to a quarter cycle (90°) are multiplied by present current values to obtain reactive power. Active–reactive metering of this type is a widely utilized method for determining displacement power factor for utility billing. These meters do not accurately measure the effect of harmonic currents because the delay of the voltage samples is based on the fundamental frequency and is incorrect for the harmonics. (The important 5th harmonic, which is frequently the predominant harmonic component, is accurately measured because it is delayed 450° ($5 \times 90^\circ$), which is equivalent to the correct 90° delay).

Metering for Nonlinear Loads

With the application of high-speed digital computing techniques to measurement of ac currents and voltages, together with digital filtering, the quantities necessary for accurate and correct calculation of power factor are susceptible to direct computation. In practice, the ac waveforms are sampled at a frequency greater than twice the highest frequency to be measured, in compliance with well-known sampling theories. Data obtained can be treated using Fourier's equations to calculate rms values for voltage, current, and phase angle for the fundamental and each harmonic frequency. Power quantities can be obtained with digital filtering in strict accordance with their *ANSI/IEEE STD 100* definitions. Power quantities can be displayed for the fundamental only (displacement power factor), or for fundamental plus all harmonics (power factor for nonsinusoidal waveforms).

Metering Applications

Instruments with the capabilities described above are often called *harmonic analyzers*, and are available in both single-phase and polyphase versions. They can be portable, in which case they are often used for power surveys, or panel mounted for utility and industrial revenue metering. Polyphase analyzers can be connected as shown in Figures 40.13 and 40.14. Care must be taken to connect the instrument properly. Both voltage leads and current transformers must be connected to the proper phases, and the current transformers must also be placed with the correct polarity. Most instruments use color-coded voltage connectors. Correct polarity is indicated on the current transformers by arrows or symbols, and complete

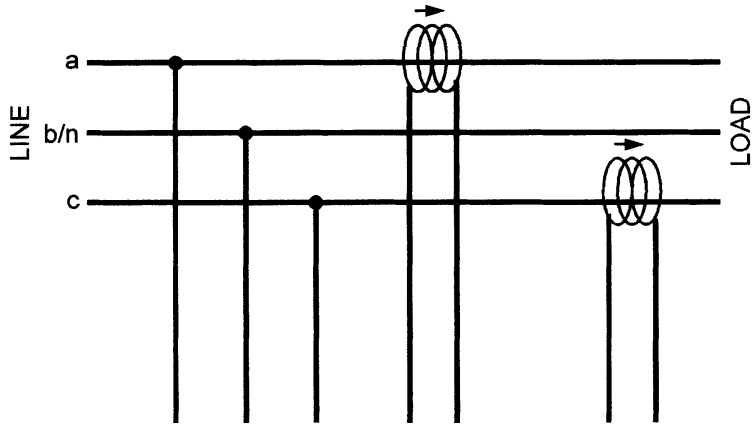


FIGURE 40.13 Meter connections for a one-, two-, or three-phase, three-wire service. Voltage leads are connected to phases a, b, and c, or to a, c, and neutral as the system dictates. Care must be taken to connect the voltage lead for each phase to the input corresponding to the input for the current transformer reading that phase, and directional characteristics of the transformers must be observed. Only two current connections are required.

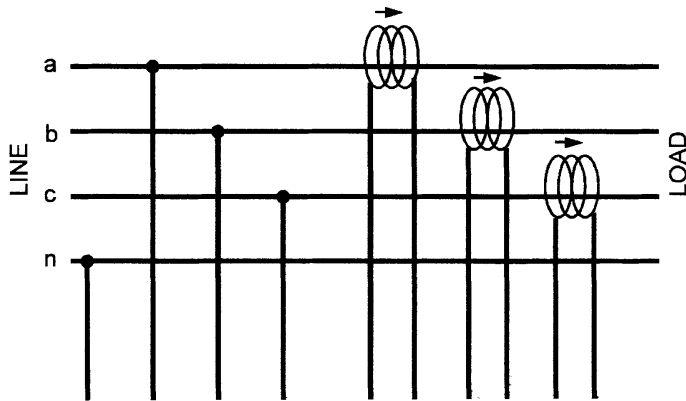


FIGURE 40.14 Meter connections for a three-phase, four-wire wye or delta connected service. Voltage leads are connected to phases a, b, and c, and to the neutral. Care must be taken to connect the voltage lead for each phase to the input corresponding to the input for the current transformer reading that phase, and directional characteristics of the transformers must be observed. In situations where measurement of neutral currents is desired, a fourth current transformer can be used to read the neutral. A fourth voltage connection might be used to read neutral-ground voltages.

hook-up and operating instructions are included. When single-phase instruments are used, the same precautions must be followed for making connections.

40.5 Instrumentation

Table 40.2 lists a sampling of harmonic and power factor measuring instruments available from major manufacturers. All instruments listed use some type of Fourier calculations and/or digital filtering to determine power values in accordance with accepted definitions. Many can be configured to measure non-harmonic-related power quality concerns. Unless otherwise noted, all instruments require the purchase of one current probe per input. Probes are available for measuring currents from 5 A to several thousand amperes. For comparison purposes, priced probes will be those with a 600-A range. Voltage leads are usually supplied as standard equipment. Table 40.3 contains addresses of these manufacturers.

TABLE 40.2 Selected Instrumentation for Harmonic and Power Factor Measurement

Manufacturer	Model	V/I inputs	Display type	Communication	Special features	List price (US\$)
Hand-held units						
Amprobe	HA2000 1/1	Visual	RS232	Hand-held, probe included, 21 nonvolatile memories	990	
BMI	155	1/1	Visual	Optional printer, RS232	Hand-held, 1145 + 550 (printer) + 380 (probe)	
BMI	355	4/3	Visual	Optional printer, RS232	Hand-held	1995 + 550 (printer) + 380/probe
Dranetz 4300	4/4	Panel for data and graphs RS232	Hand-held, battery or ac power, optional PCMCIA memory card	5000 + 450/probe		
Fluke	39	1/1	Visual	None	Probe included	895
Fluke	41b	1/1	Visual	RS232	Probe included, 8 memories, logging with computer and supplied software	1795
Portable units						
BMI	3030A	4/4	Built-in printer for data and graphs optional internal modem	Portable, optional built-in disk drive for storage, long-term monitoring, optional PQ configurations	6800 + 600 (modem) + 1895 (disk storage) + 590/probe	
Dranetz	PP1-R	4/4	Panel for data and graphs	PCMCIA card slot	Long-term monitoring, optional PQ configurations, remote control by software	12,000 + 450/probe
PC-based units						
BMI	7100	4/4	PC-based (not included)	PC connection cord	Portable, uses computer for storage, optional software, long-term monitoring, optional PQ configurations	5294 + PC + 395 (software) + 415/probe
Cooper	V-Monitor II	4/4	PC-based (not included)	Serial port	Portable, software and signal interface and data acquisition board, long-term monitoring	12,000 + 500/probe

TABLE 40.2 (continued) Selected Instrumentation for Harmonic and Power Factor Measurement

Manufacturer	Model	V/I inputs	Display type	Communication	Special features	List price (US\$)
RPM	1650	4/5	PC-based, not included	Ethernet long-term monitoring, optional PQ configurations, remote control by software	6250 + 750 software + 490/probe	
Panel-mounted						
Cutler-Hammer/ Westinghouse General Electric	4/4 kV	Panel Vector Electricity Meter	Optional IMPACC Socket	Panel-mount to replace meters, monitoring Programmable multifunction LCD display pulse output for measured power quantities replaces industrial revenue meters, calculates and accumulates all power and revenue data	3690 + input devices 595 + 495 (software)	
Square D	Powerlogic PM620	3/3	LCD panel	RS485	Panel meter replacement, calculates and accumulates power data	1583 + probes
Square D	Powerlogic CM2350	3/3	Panel	RS485	Connect to network, remote controller, monitoring	4290 + probes

TABLE 40.3 Manufacturers of Power Factor Measuring Harmonic Analyzers

Amprobe Instruments 630 Merrick Road Lynbrook, NY 11563 Tel: (516) 593-5600	Cutler Hammer Inc. Westinghouse & Cutler-Hammer Products Five Parkway Center Pittsburgh, PA 15220 Tel: (412) 937-6100	GE Meter 130 Main Street Somersworth, NH 03878 Tel: (603) 749-8477
BMI 3250 Jay Street Santa Clara, CA 95054 Tel: (408) 970-3700	Dranetz Technologies, Inc. 1000 New Durham Road Edison, NJ 08818-4019 Tel: (800) DRANTEC	Reliable Power Meters, Inc. 400 Blossom Hill Road Los Gatos, CA 95032 Tel: (408) 358-5100
Cooper Power Systems Division 11131 Adams Road P.O. Box 100 Franksville, WI 53126-0100 Tel: (414) 835-2921	Fluke Corporation P.O. Box 9090 Everett, WA 98206 Tel: (800) 44FLUKE	Square D Power Logic 330 Weakley Road Smyrna, TN 37167-9969 Tel: (615) 459-8552

Defining Terms

Active power: A term used to express the real power delivered to do work in an ac distribution system.

Reactive power: A term used to express the imaginary power that does no work but provides magnetization to enable work in an ac distribution system.

Phasor power: A term used to express the product of volts and amperes in an ac distribution system in which voltage and current are sinusoidal.

Harmonic power: A term used to express the power due to harmonic frequencies in an ac distribution system in which voltage and/or current are nonsinusoidal.

Apparent power: A term used to express the product of volts and amperes in an ac distribution system in which voltage and/or current are nonsinusoidal.

Power factor: A single number, calculated by dividing active power by either the phasor power or the apparent power, which describes the effective utilization of ac distribution system capacity.

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Further Information

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