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Q Factor Measurement

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Q factor is a method of characterizing the rate of dissipation of energy from an oscillating system. Q is defined as 2π times the energy stored in a resonant system divided by the energy dissipated per cycle. The term system used in this context refers to any type of resonance: mechanical, electrical, nuclear, etc. For the purposes of this *Handbook*, Q will be that of an electric circuit. Also, for the discussion of Q, very low values of Q, typically less than 10, will not be considered as these low values of Q produce highly damped oscillations that are more exponential than oscillatory and the concept of Q does not fit.

A common interpretation of Q is quality factor that explains the use of the letter, Q, but this is misleading. A component that has a high Q is not always beneficial and may not be of high quality. In many circuits, a component requires a specific value of Q rather than "higher is better." In other cases, a high Q is an advantage.

The Q factors encountered in common circuit components range from a low of about 50 for many inductors to nearly 1 million found in high-quality quartz crystals. Q can be applied to a resonant circuit or to capacitors and inductors. When Q is applied to a component, such as an inductor, the Q would be that obtained if the inductor were used in a resonant circuit with a capacitor that dissipates no energy. In this case, the value of Q depends on the frequency.

For most LC resonant circuits, the losses in the inductor dominate and the Q of the inductor is, essentially, the Q of the circuit. It is easy to make very low-loss capacitors even in the UHF region. On the other hand, varactor diodes have considerably more loss than fixed capacitors, and a varactor can play a more significant role in setting the circuit Q.

52.1 Basic Calculation of Q

Figure 52.1 shows a simple resonant circuit. The capacitor can store energy in the electric field and the inductor in the magnetic field. The circuit oscillates with the energy transferring between the two elements. For the ideal elements shown, this continues forever. Since the definition of Q has the energy lost per cycle in the denominator — which is zero — the result is an infinite Q.

In practice, circuit elements are not perfect and the energy initially contained within this circuit would be lost by the circuit and the oscillations would decrease in amplitude as the energy diminished. Energy loss in a circuit is represented by that in a resistor which can be included in one of two ways. The first

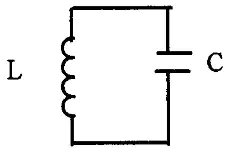

FIGURE 52.1 A simple resonant circuit with no loss elements.

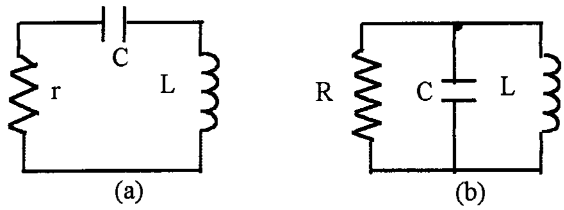

FIGURE 52.2 (a) A simple series resonant circuit with the equivalent resistance. (b) A parallel resonant circuit with a parallel equivalent resistance. For the same *Q* circuit, the values of the resistors are not the same.

way is shown in Figure 52.2(a), where the resistor is in series with the capacitor and inductor. A second representation is a parallel resistor as shown in Figure 52.2(b).

To derive the relationship between the circuit element values and the *Q* of the circuit, either the current or voltage of the circuit can be used in the equations. Current is the obvious choice for a series circuit, while voltage is the common thread for the parallel circuit. Assume that the amplitude of the current through the series circuit of Figure 52.2(a) is given by:

$$i(t) = I(t)\cos(2\pi f_0 t) \tag{52.1}$$

where f_0 = Resonant frequency = $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$ (52.2)

and $I(t)$ is the amplitude, which is decreasing in some unspecified fashion. The circuit's peak current occurs when the cosine function is equal to 1 and all of the energy is contained in the inductor and is equal to $(1/2)LI^2(t)$.

Assume that a relatively high *Q* is present in this circuit and that $I(t)$ changes by only a slight amount during the time of one cycle. During this cycle, the peak current is $I(t)$, and the rms value of the current is $(0.707)I(t)$. Therefore, the energy dissipated in one cycle is $(0.5)I^2(t)r/f_0$. Substituting these values in the definition of *Q* yields:

$$Q = 2\pi \frac{\frac{1}{2}LI^2(t)}{\frac{1}{2}\frac{rI^2(t)}{f_0}} = \frac{2\pi f_0 L}{r} = \frac{X_L}{r} \tag{52.3}$$

where X_L is the reactance of the inductor. The same procedure can be used with the parallel resonant circuit of Figure 52.2(b) using voltage equations to obtain the relationship between a parallel resistance and *Q*, which is:

$$Q = \frac{R}{X_L} \tag{52.4}$$

It is very important to understand the nature of the circuit resistance in Figure 52.2(a) and (b). This resistance represents all of the losses in the resonant circuit. These losses are from a variety of sources,

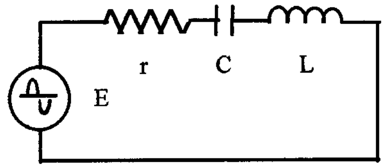

FIGURE 52.3 A series resonant circuit showing a driving source.

such as the resistance of the wire to make an inductor or the leakage current of a capacitor. It can also represent the deliberate addition of resistors to set the Q of the circuit to a specific value. The resistance shown in the circuits of Figure 52.2 represents the equivalent resistance of all of the energy losses. This resistance cannot be measured with an ohmeter as the value of the equivalent resistor is a function of frequency and other variables such as signal level. Some of the loss in a resonant circuit is due to radiation, which is a function of frequency. The resistance of conductors is mostly due to skin effect, which increases with increasing frequency. The losses in the ferromagnetic materials used for making some inductors are nonlinear; thus, the equivalent resistance is not only a function of frequency but of signal level.

Most resonant circuits are not stand-alone circuits as shown in Figure 52.2, but are a part of other circuits where there are sources and loads. These additional resistances further remove energy from the resonant circuit. The Q of a resonant circuit when there are sources and loads is called the *loaded Q*. In most applications of resonant circuits, the Q of the resonance is set by the external loads rather than the capacitor and inductor that form the resonance.

52.2 Bandwidth and *Q*

The *bandwidth* of a resonant circuit is a measure of how well a resonant circuit responds to driving signals of a frequency near the resonant frequency and is a function of Q. The relationship between the 3 dB bandwidth and Q will be derived.

Applying a driving signal to a resonant circuit can overcome the losses of the circuit and cause the resonant circuit to oscillate indefinitely. As an example of this, consider the voltage generator in the series resonant circuit shown in Figure 52.3.

When the frequency of the voltage source is equal to the resonant frequency of the circuit, the equivalent impedance of the series resonant circuit is the resistance of the circuit and the current in the circuit, simply E/r.

At frequencies higher or lower than the resonant frequency, the impedance of the circuit is greater because the net reactance is not zero and the circuit current will be less than at resonance.

At what frequency will the circuit current be 3 dB less than at resonance? This frequency is where the circuit impedance is 1.414 that of the impedance at resonance. This is the frequency where the reactive part of the impedance is equal to the real part. This situation occurs at two frequencies. Below the resonant frequency, the net reactance is capacitive and is equal to r, while at a second frequency above the resonant frequency, the reactance is inductive and equal to r. This can be represented by two equations for the two frequencies:

For $f_1 > f_0$,
$$\left|X_L - X_C\right| = 2\pi f_1 L - \frac{1}{2\pi f_1 C} = \frac{\left(\dfrac{f_1}{f_0}\right)^2 - 1}{2\pi f_1 C} = r \tag{52.5}$$

$$\left(\frac{f_1}{f_0}\right)^2 - \frac{1}{Q}\left(\frac{f_1}{f_0}\right) - 1 = 0 \qquad \left(\frac{f_1}{f_0}\right) = \frac{1}{2Q} \pm \sqrt{\frac{1}{4Q^2} + 1}$$

For $f_2 < f_0$,

$$\left|X_L - X_C\right| = \frac{1}{2\pi f_2 C} - 2\pi f_2 L = \frac{-\left(\frac{f_1}{f_0}\right)^2 - 1}{2\pi f_2 C} = r \qquad (52.6)$$

$$\left(\frac{f_2}{f_0}\right)^2 + \frac{1}{Q}\left(\frac{f_2}{f_0}\right) - 1 = 0 \qquad \left(\frac{f_2}{f_0}\right) = \frac{1}{2Q} \pm \sqrt{\frac{1}{4Q^2} + 1}$$

$$f_1 - f_2 = \frac{f_0}{Q} = \text{bandwidth}$$

Measuring *Q*

There are a number of methods of measuring Q using a variety of bridges, several of which are described in [1]. One method of measuring a capacitive or inductive Q is to place the component in a resonant circuit. When the Q to be measured of a device that is, in itself, a resonant circuit such as quartz crystal, similar techniques are used except the device's own resonance is used for the measurement. Circuit voltages or currents are measured at the resonance frequency and the Q is determined.

52.3 The *Q*-Meter

One simple and very popular method of measuring Q is with a device called, appropriately, the Q-meter. Consider the resonant circuit in Figure 52.4 for measuring the Q of inductors. This circuit has a very low-loss capacitor of known value and a constant voltage source.

The usual components measured by the Q-meter are inductors. It was previously mentioned that inductors are the weak link in resonant circuits, and the Q of a circuit is usually set by the inductor. The Q-meter can measure capacitance and capacitor Q. In this theoretical circuit, the circuit resistance is the equivalent series resistance of the inductor under test. This is due to the fact the variable capacitor is assumed to be lossless, the generator has zero resistance, and the voltmeter does not appreciably load the circuit. In a real circuit, it is not possible to achieve this situation, but the instrument is designed to approach this goal.

To measure the Q of an inductor using the Q-meter, the generator is set to the desired frequency while the variable capacitor tunes the circuit to resonance as indicated by the peak reading of the voltmeter.

At resonance, the impedance of the circuit is simply the equivalent series resistance of the inductor. This sets the current of the circuit as:

$$I = E/R_X \qquad (52.7)$$

where E is the generator voltage and R_X is the equivalent resistance of the inductor.

Because the circuit is at resonance, the voltages of the two reactances are equal and of opposite phase. Those voltages are:

$$V = IX_C \quad \text{or} \quad V = IX_L \qquad (52.8)$$

where X_C is the capacitive reactance and X_L is the inductive reactance, which are numerically equal at resonance.

Substituting the relationship of the circuit current, the result is:

$$V = EX_L/R_X = EX_C/R_X = EQ \qquad (52.9)$$

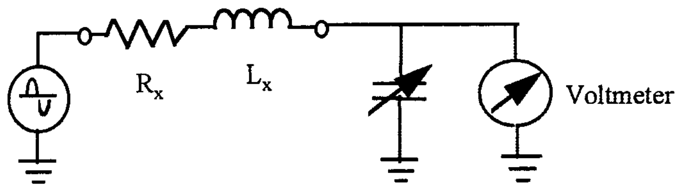

FIGURE 52.4 The basic circuit of a *Q*-meter showing the signal source, the inductor under test, and the voltmeter.

Therefore, the voltage across the reactances is equal to *Q* times the applied voltage. If, as an example, the voltage source were 1 V, the voltmeter would read *Q* directly. *Q* values of several hundred are common and, therefore, voltages of several hundred volts could be measured. Modern circuits do not typically encounter voltages of this magnitude, and many components cannot withstand this potential. Therefore, most *Q*-meters use a much smaller source voltage, typically 20 mV.

If the frequency of the source and the circuit capacitance are known, it is possible to calculate the inductance of the unknown.

52.4 Other *Q* Measuring Techniques

There are very few *Q*-meters being manufactured today, although there are thousands of old *Q*-meters still in use. Because the *Q*-meter was the only accepted method of measuring *Q* for so many years, it will take decades before alternative methodologies overtake the *Q*-meter.

Measuring *Q* without a *Q*-meter involves a variety of *RLC* measuring instruments that measure the vector impedance and calculate *Q*. The calculated *Q* value is not as valid as that determined with a *Q*-meter unless the *RLC* meter is capable of measuring *Q* at the desired frequency. Only the more sophisticated, and expensive, *RLC* meters allow the use of any test frequency. Despite the new sophisticated *RLC* measuring instruments, there is an adapter for one model *RLC* instrument that allows the classic *Q*-meter-style measurement to be made.

Because vector impedance measurement is covered elsewhere in this Handbook, the remainder of this section will be devoted to measurements using the *Q*-meter.

52.5 Measuring Parameters Other than *Q*

In addition to *Q*, the *Q*-meter can be used to measure inductance, the *Q* or dissipation factor of a capacitor, and the distributed capacitance, C_d, of an inductor.

If an inductor with capacitance C_d is placed in the *Q*-meter circuit, the total circuit capacitance includes both the *Q*-meter's capacitance plus the additional C_d. Therefore, when resonance is achieved, the actual resonating capacitance is more than what is indicated on the *Q*-meter capacitor's dial. If C_d is not included in the calculation of inductance, the resulting value would be too large.

In many applications, the actual inductance is not the important parameter to be measured by the *Q*-meter. The actual parameter being determined is "what capacitance is required to resonate the particular inductor at a specific frequency," regardless of C_d.

In other applications, such as inductors that are to be used in wide-range oscillators, where C_d will limit the tuning range, C_d is an important parameter.

The *Q*-meter can be used to determine C_d, which will also allow for an accurate calculation of inductance. Determining C_d is a matter of resonating the inductor under test at more than one frequency.

To understand how the two-frequency measurement will allow the determination of C_d, assume an inductor is resonated at a frequency f_1. The relationship between the applied frequency, f_1, and the capacitor of the *Q*-meter to obtain resonance is:

$$f_1 = \frac{1}{2\pi\sqrt{L\left(C_1 + C_d\right)}} \tag{52.10}$$

where C_1 is the capacitance set on the Q-meter.

Resonating the same inductor at a second, higher, frequency, f_2, requires a Q-meter capacitance of C_2 such that:

$$f_2 = \frac{1}{2\pi\sqrt{L\left(C_2 + C_d\right)}} \tag{52.11}$$

This implies that C_2 is a smaller capacitance than C_1. Using these two equations and solving for C_d, the following result is obtained.

$$C_d = \frac{C_2 f_2^2 - C_1 f_1^2}{f_1^2 - f_2^2} \tag{52.12}$$

A convenient relationship between f_1 and f_2 is to set $f_2 = 1.414\ f_1$. With frequencies thus related, the distributed capacitance is:

$$C_d = C_1 - 2C_2 \tag{52.13}$$

C_d causes errors in the measurement of Q because of current through the C_d. The Q measured by the Q-meter is called the "effective Q." Since large inductors with significant C_d are no longer in common use because of the use of active filters, the distinction between effective Q and real Q is seldom considered. For additional information about effective Q and distributed capacitance, see [2].

To measure capacitors on the Q-meter, a relatively high Q inductor is connected to the inductance terminals on the Q-meter and resonated. The capacitor to be measured is connected to the capacitor terminals, which increases the circuit capacitance. The Q-meter variable capacitor is adjusted to regain resonance, which requires that the capacitance be reduced by an amount equal to the unknown capacitor.

The Q of the capacitor can be measured. The addition of the capacitor reduces the circuit Q because of the additional loss introduced by the capacitor. In the description of the Q-meter, the Q-meter's variable capacitor is assumed to have no loss.

Measuring the Q of a capacitor using the Q-meter is seldom done. This is because most capacitors have very high Q values. There are special cases, such as measuring the Q of a transducer or a varactor diode, where low Q values are encountered.

The unknown capacitor is connected to the CAP terminals of the Q-meter, which are simply in parallel with the Q-meter's variable capacitor. The variable capacitor in the Q-meter is set to the minimum capacitance and a suitable inductor is placed across the IND (inductor) terminals. The Q-meter is resonated using the frequency control rather than the variable capacitance.

For best results, the Q of the inductor must be considerably greater than that of the unknown capacitor, and the unknown capacitance must be considerably greater than the internal capacitance. If these criteria are met, the Q of the unknown capacitor can be read from the Q-meter. If these criteria are compromised, corrections can be made but the equations become complex and the accuracy degrades.

Most Q-meters provide measurement ranges to about 500 or to 1000. This is sufficient for measuring inductors, which was the main purpose of the Q-meter. For measuring high-Q devices such as ceramic resonators with Q values greater than 1000 or for quartz resonators with Q values extending into the hundreds of thousands, the Q-meter technique is insufficient. A high-Q circuit implies very little energy loss, and the energy that must be removed to make a measurement must be very small if Q is to be measured accurately.

Defining Terms

Bandwidth: A measurement of the amount of frequency spectrum occupied by a signal or the equivalent spectrum covered by a circuit that passes a finite band of frequencies. There are a number of methods of defining bandwidth, depending on the nature of the spectrum. Relative to resonant circuits, the bandwidth is measured between the –3 dB points of the passband.

Distributed capacitance: The amount of capacitance added to an inductor typically from the capacitance due to adjacent wires in a solenoid-type inductor. The distributed capacitance is given as a single capacitance figure for a specific inductor and can be defined as the equivalent capacitance across the entire coil. This would also allow the inductor to have a self-resonant frequency where the inductor resonates with the distributed capacitance with no external capacitance.

Effective inductance: Due to distributed capacitance, less capacitance than that calculated from an inductance value is required to resonate a circuit. If the actual capacitance required to resonate a circuit is used to calculate an inductance value, the resulting inductance value will be higher than the theoretical inductance value. This higher value is called the "effective inductance." The actual inductor cannot be considered as a pure inductance of a value equal to the effective inductance because the real inductor has a resonant frequency that a pure inductance does not.

***Q*:** A measurement of the rate of energy loss in a resonant circuit.

***Q*-Meter:** An instrument for measuring *Q* factor by resonating the circuit and measuring the voltage across the reactances.

References

1. *Reference Data for Engineers: Radio Electronics, Computer and Communications, 8th ed.*, Sams, Englewood Cliffs, NJ: Prentice-Hall Computer Publishing, 1993.
2. A. Helfrick and W. Cooper, *Modern Instrumentation and Measurement Techniques,* Englewood Cliffs, NJ: Prentice-Hall, 1990.